

Geometry I, 2006 : Coursework 7 Solutions

1. [1 mark for correct A ; 1 mark for correct B]

$$A = \begin{pmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \end{pmatrix}$$

$$B = \begin{pmatrix} 3 & 3 \\ 5 & 5 \\ 7 & 7 \end{pmatrix}$$

2. (i) [1 mark; justification required]

Yes. For example, take $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$. Then $A \neq 0_{22}$, but $A^2 = 0_{22}$.

- (ii) [1 mark; justification required]

Yes. For example, take $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Then $AB = 0_{22}$, but $BA = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

3. Let $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, $X = (x_{ij})_{n \times p}$.

(i) [2 marks] The (i, j) -entry of the $m \times n$ matrix $A + B$ is $a_{ij} + b_{ij}$, which is equal to $b_{ij} + a_{ij}$, the (i, j) -entry of the $m \times n$ matrix $B + A$. Hence $A + B = B + A$.

(ii) [1 mark] The (i, j) -entry of the $m \times n$ matrix 0_{mn} is 0, and so the (i, j) -entry of the $m \times n$ matrix $0_{mn} + A$ is $0 + a_{ij} = a_{ij}$. Hence, $0_{mn} + A = A$.

(iii) [1 mark] The (i, j) -entry of the $m \times n$ matrix $-A$ is $-a_{ij}$, and so the (i, j) -entry of the $m \times n$ matrix $A + (-A)$ is $a_{ij} - a_{ij} = 0$. Hence, $A + (-A) = 0_{mn}$.

(iv) [2 marks] The (i, j) -entry of the $m \times n$ matrix $(\alpha + \beta)A$ is $(\alpha + \beta)a_{ij} = \alpha a_{ij} + \beta a_{ij} =$ the (i, j) -entry of αA + the (i, j) -entry of $\beta A =$ the (i, j) -entry of the $m \times n$ matrix $\alpha A + \beta A$. Hence $(\alpha + \beta)A = \alpha A + \beta A$.

(v) We first remark that all three matrix expressions to be proved equal have the same size ($m \times p$).

[1 mark] The (i, j) -entry of $\alpha(AX)$ is $\alpha(a_{i1}x_{1j} + a_{i2}x_{2j} + \cdots + a_{in}x_{nj})$.

[1 mark] The (i, j) -entry of $(\alpha A)X$ is $(\alpha a_{i1})x_{1j} + (\alpha a_{i2})x_{2j} + \cdots + (\alpha a_{in})x_{nj}$.

[1 mark] The (i, j) -entry of $A(\alpha X)$ is $a_{i1}(\alpha x_{1j}) + a_{i2}(\alpha x_{2j}) + \cdots + a_{in}(\alpha x_{nj})$.

[1 mark] In all three cases, the (i, j) -entry is equal to $\alpha a_{i1}x_{1j} + \alpha a_{i2}x_{2j} + \cdots + \alpha a_{in}x_{nj}$, and so the three matrices (of the same size) are equal.

4. [1 mark each]

(i) $\begin{pmatrix} 2 & 6 & 7 \\ 1 & 5 & 7 \\ -7 & 1 & -2 \end{pmatrix}$

(ii) This has no meaning since the size of A (3×3) is different than the size of B (3×2).

(iii) $\begin{pmatrix} 1 & 9 & -3 \\ 6 & -2 & 12 \\ -21 & 3 & 13 \end{pmatrix}$

(iv) $\begin{pmatrix} -5 & 2 \\ -4 & 16 \\ 33 & 2 \end{pmatrix}$

(v) This has no meaning since the number of columns of B is not equal to the number of rows of C .

(vi) Note: to save work, can calculate as $C(AB)$.

$$\begin{pmatrix} 247 & 66 \\ 84 & 84 \\ -231 & -14 \end{pmatrix}$$

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